http://mail.bracuniversity.net/openwebmail/images/openwebmail.gif

# Probability: Introduction to Basic Concept

The word ***probability*** is a commonly used term that relates to the chance that a particular event will occur when some experiment is performed. Uncertainty pervades all aspects of human endeavor. Probability is one of our most important conceptual tools because we use it to assess degrees of uncertainty and thereby to reduce risk. Whether or not one has had formal instruction in this topic, s/he is already familiar with the concept of probability since it pervades almost all aspects of our lives. Without consciously realizing it many of our decisions are based on probability. For example, when y ou study for an examination, you concentrate more on areas that you feel are likely to be covered on the test. You may cancel or postpone an outdoor activity if you believe the likelihood of rain is high.

In business, probability plays a key role in decision-making. The owner of a retail shoe store, for example, orders heavily in those sizes that s/he believes likely to sell fast. The owner of a movie theatre schedules matinees only during holiday seasons because the chances of filling the theatre are greater at that time. The two companies decide to merge when they believe the probability of success is greater for the consolidated company than for either independently.

**Some important Definitions:**

**Experiment:**

An experiment is any process that produces an observation or outcome. Experiment is an act that can be repeated under given conditions.

Experiment can be of two types –

1. *Deterministic experiment*
2. *Random experiment*

Usually, the exact result of the experiment cannot be predicted with certainly.

Unit experiment is known as *trial*. This means that trial is a special case of experiment. Experiment may be a trial or two or more trials.

Example: some example of experiments and their sample spaces are as follows.

**Sample space & Sample point:**

A sample space of an experiment is a set or collection of all possible outcomes of the same experiment and is usually denoted by the symbol.

Each outcome of an experiment can be thought of as a sample point or element in the sample space.

**Example:**

Some example of experiments and their sample spaces are as follows.

1. If the experiment consists of flipping two coins and noting whether they land heads or tails, then
2. If the outcome of the experiment is the gender of a child, then

Where outcome *G* means that the child is a girl and *B* that it is a boy.

1. Consider an experiment that consists of rolling two balanced dice, one black and one red are thrown and number of dots on their upper faces are noted, also if b be the outcomes of the black die and r be the outcomes of the red die. If we let denote the outcome in which black dice has value *b* and red dice has value *r*, then the sample space of this experiment is:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| ⇒ | 1 | 2 | 3 | 4 | 5 | 6 |
| ⇓ |
| 1 | 1,1 | 1,2 | 1,3 | 1,4 | 1,5 | 1,6 |
| 2 | 2,1 |  |  |  |  |  |
| 3 |  |  |  |  | 3,5 |  |
| 4 |  |  |  |  |  |  |
| 5 | 5,1 |  |  |  |  |  |
| 6 | 6,1 | 6,2 |  |  |  | 6,6 |

**Events:**

Any set of outcomes of the experiment is called an event. One or more outcomes of an experiment constitute an event.

Events are generally denoted by capital letter A, B, C, etc.

There are two types of events

**Simple event -** … if an event contains only one sample point.

**Compound event -** if an event contains more than one sample point.

**Different approaches in defining probability:**

There are different approaches in defining probability are:

1. Classical or Mathematical or a priori probability
2. Empirical or frequency probability
3. Subjective probability
4. **Classical or Mathematical or a priori probability:**

If there are n mutually exclusive, equally likely and exhaustive outcomes of an experiment and if m of this outcomes are favorable to an event A, then the probability of the event A which is denoted by P(A) is defined by

|  |  |
| --- | --- |
|  | Favorable outcomes of an event A  Total number of outcomes of the experiment |

There are three drawbacks of classical definition of probability:

* 1. The classical probability fails to define probability when the total number of possible outcomes is infinite.
  2. The classical definition leaves us completely helpless when the possible outcomes are not equally likely.
  3. It is not always possible to enumerate all the equally likely cases.

1. **Empirical or frequency probability:**

If an experiment is repeated a large number of times (n times) under the same conditions and an event A occurs m times then according to empirical or frequency approach of probability the probability of the event A

|  |  |
| --- | --- |
|  | Number of times event A occurs.  Total number of trials. |

Drawbacks of the empirical probability:

1. In practice, it is not possible to repeat the experiment an infinite number of times under the same conditions to get the probability.
2. It is not clear how large n should be before we are certain that the probability, p is close to the limiting of  as .
3. **Subjective probability:**

The probability that a person assigns to an event on the basis of his own judgment, beliefs and information about the event is known as subjective probability.

The subjective probability has the following drawbacks,

1. It varies from individual to individual as it depends on individual’s judgment and belief,
2. It has no objective basis.

**Problem:**

A bag contains 4 white and 6 black balls. If one ball is drawn at random from the bag, what is the probability that it is i. Black, ii. White, iii. White or black and iv. Red.

**Answer:**

1. Let A be the event that the ball is black, then the number of outcomes favorable to A is 6. Hence

|  |  |
| --- | --- |
|  | Favorable outcomes of an event A = Number of black balls  Total number of outcomes of the experiment = Total number of balls |

1. Let B be the event that the ball is white, and then the number of outcomes favorable to B is 4. Hence

|  |  |
| --- | --- |
|  | Favorable outcomes of an event B = Number of white balls  Total number of outcomes of the experiment = Total number of balls |

1. Let C be the event that the ball is black or white and then the number of outcomes favorable to C is 10. Hence

|  |  |
| --- | --- |
|  | Favorable outcomes of an event C = Number of white or white balls  Total number of outcomes of the experiment = Total number of balls |

1. Let D be the event that the ball is red, and then the number of outcomes favorable to B is 4. Hence

|  |  |
| --- | --- |
|  | Favorable outcomes of an event D = Number of red balls  Total number of outcomes of the experiment = Total number of balls |

**Problem:**

A card is drawn from a pack of 52 cards. Find the probability that it is i. A red card, ii. A spade, iii. An ace, iv. Not a spade and v. a king or a queen.

**Answer:**

When a card is drawn from a pack of 52 cards, the total number of equally likely and mutually exclusive outcomes are 52. That is here 

1. Let A be the event drawing a red card. There are 26 black card and 26 red cards in a pack and any one of the red cards can be drawn in 26 ways. Hence

|  |  |
| --- | --- |
|  | Favorable outcomes of an event A = Number of red cards balls  Total number of outcomes of the experiment = Total number of cards |

1. Let B be the event drawing a spade. There are 13 spades. Hence

|  |  |
| --- | --- |
|  | Favorable outcomes of an event B = Number of spades  Total number of outcomes of the experiment = Total number of cards |

1. Let C be the event drawing an ace. There are 4 spades. Hence

|  |  |
| --- | --- |
|  | Favorable outcomes of an event C = Number of ace  Total number of outcomes of the experiment = Total number of cards |

1. Let D be the event drawing a card that is not a spade. There are 39 cards that is not spade. Hence

|  |  |
| --- | --- |
|  | Favorable outcomes of an event D = Number of cards not spade  Total number of outcomes of the experiment = Total number of cards |

1. Let E be the event drawing a card will be either king or queen. There are 4 kings and 4 queens. Hence

|  |  |
| --- | --- |
|  | Favorable outcomes of an event E = Number of kings and queens  Total number of outcomes of the experiment = Total number of cards |

**Some basic event operations:**

|  |  |
| --- | --- |
| **Shaded region is Complement of A** | **Complement of any event:**  For any event *A,* the complement of A (denoted by or or) with respect to  is the set of all elements that are in  but not in A. |
| **Shaded region is** | **Union of events:**  For any two events *A* and *B,* the *Union of events A* and *B* is the set of all elements that are in *A* or in *B* or in both *A* and *B*.  Let  and  , .  Then the union of the two sets denoted by. |
| **Shaded region is** | **Intersection of sets:**  For any two events *A* and *B,* the *intersection* *of events A* and *B* is the set of elements that are both in *A* and *B*.  Let  and  , .  Then the intersection of the two sets denoted by. |
| *A* *B*              ***A* and *B* are disjoint** | **Disjoint or mutually exclusive sets:**  If A and B be two subsets of , then A and B are said to be disjoint or mutually exclusivesets if they have no elements in common. That is, if.  Let  and, . Then the union of the two sets denoted by. |

**Sampling with replacement:**

If the elements of a sample are drawn randomly one by one and after each draw the element is returned to the population then the drawing is said to be done with replacement and the process of having the sample is called random sampling with replacement.

**Sampling without replacement:**

If the elements of a sample are drawn randomly one by one and after each draw the element is not returned to the population then the drawing is said to be done without replacement and the process of having the sample is called random sampling with replacement.

**Problem:**

A box contains three balls – one red, one blue and one yellow. Consider an experiment that consists of drawing a ball from the box, replacing it and withdrawing a second ball.

1. What is the sample space of this experiment?
2. What is the event that the first ball drawn is yellow?
3. What is the event that the same ball is drawn twice?

**Problem:**

Repeat the previous problem when the second ball is drawn without replacement of the first ball.

**Problem:**

An experiment consists of flipping a coin 3 times and each time noting whether it lands heads or tails.

1. What is the sample space of this experiment?
2. What is the event that tails occur more often than heads?

**Problem:**

Let, , and find

|  |  |
| --- | --- |
|  |  |
|  |  |

**Problem:**

Two balanced dice, one black and one red are thrown and the number of dots on their upper faces are noted, let b be the outcomes of the black die and r be the outcomes of the red die. Now answer the following:

* 1. List a sample space of the experiment.
  2. What is the probability of throwing a double?
  3. What is the probability that the sum is 5, that is?
  4. What is the probability that the sum is even?
  5. What is the probability that  or?
  6. What is the probability that the number on the red die is at least 4 greater than the number on the black dice.

**Answer:**

1. If two balanced dice, one black and one red are thrown and the number of dots on their upper faces are noted, also if b be the outcomes of the black die and r be the out comes of the red die. Then the sample space for the given experiment will be as follows:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| ⇒ | 1 | 2 | 3 | 4 | 5 | 6 |
| ⇓ |
| 1 | 1,1 | 1,2 | 1,3 |  |  | 1,6 |
| 2 | 2,1 |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 | 6,1 | 6,2 |  |  |  | 6,6 |

1. Let the event A = {the two dice shows the same number}

= {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}; There fore 

1. Let the event B = {The sum of the two dies is 5, that is the two dice shows the same number}

= {(1,4), (2,3), (3,2), (4,1)}

There fore 

1. Let the event C = {The sum of the two dies is even}

= {(1,1), (1,3), (1,5), (2,2), (2,4), …, … , … ,(6,4), (6,6), }

There fore 

1. Let the event D = {(b,r)| or }

= {(1,1), (1,2), (1,3), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (5,1), (5,2), (6,1), (6,2)}

There fore 

1. Let the event E = {} = {(1,5), (1,6), (2,6)}

There fore 

**Problem:**

A cafeteria offers a three – course meal. One chooses a main course, a starch and a desert. The possible choices are as follows.

|  |  |
| --- | --- |
| **Meal** | **Choices** |
| Main Course | Chicken or roast beef |
| Starch Course | Pasta or rice or potatoes |
| Dessert | Ice cream or gelatin or apple pie |

An individual is to choose one course from each category.

1. List all the outcomes in the sample space.
2. Let *A* be the event that ice cream is chosen. List all the outcomes in *A*.
3. Let *B* be the event that Chicken is chosen. List all the outcomes in *B*.
4. List all the outcomes in the event.
5. Let *C* be the event that rice is chosen. List all the outcomes in C.
6. List all the outcomes in the event

**Problem:**

Phenylketonuria is a generic disorder that produces mental retardation. About one child in every 10,000 live births in the United States has Phenylketonuria . what is the probability that the next child born in a Houston hospital has Phenylketonuria?

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**Problem:**

When typing a five – page manuscript, a certain typist makes

|  |  |
| --- | --- |
| 0 errors | With probability 0.20 |
| 1 errors | With probability 0.35 |
| 2 errors | With probability 0.25 |
| 3 errors | With probability 0.15 |
| 4 or more errors | With probability 0.05 |

If you give such a manuscript to this typist, find the probability that it will contain

1. 3 or fewer errors
2. 2 or fewer errors
3. 0 errors

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***Answer: 0.95, 0.80 and 0.20***

**Properties of Probability:**

**Property 1:**

For any event *A*, the probability of *A* is a number between 0 and 1. That is,

**Property 2:**

The probability of sample S is 1. Symbolically,

**Property 3**

The probability of disjoint event is equal to the sum of the probabilities of these events. For instance, if *A* sample S is 1. Symbolically,

**Rules of probability:**

**Addition rule:**

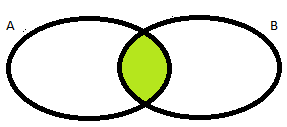
For any events *A* and *B*,



S

Note that is the probability of all outcomes that are either in *A* or in *B*. On the other hand, is the probability of all the outcomes that are in *A* plus the Probability of all the outcomes that are in *B*. Since any outcome that is in both *A* and *B* is counted twice in and only once in, it follows that

Subtracting from both sides of the preceding equation gives the addition rule.



**Problem:**

A certain retail establishment accepts either the American Express or the VISA credit. A total of 22 percent of its customers carry an American Express card, 58 percent carry a VISA credit card, and 14 percent carry both. What is the probability that a customer will have at least one of these cards?

**Solution:**

Let *A* denote the event that the customer has an American Express card

Let *B* denote the event that the customer has a VISA card.

The given information yields

,

By the additive rule, the desired probability is



That is, 66 percent of the establishment’s customers carry at least one of the cards that it will accept.

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**Problem:**

For the previous problem determine what proportion of customers has neither an American Express nor a VISA card?

**Problem:**

The family picnic scheduled for tomorrow will be postponed it is either cloudy or rainy. The weather report states that there is a 40 percent chance of rain tomorrow, a 50 percent chance of cloudiness and a 20 percent chance that it will be both cloudy and rainy. What is the probability that the picnic will be postponed?

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***Answer: 0.70***

Assignment on Probability

1. Three thousand tosses of a certain coin gave 1800 heads. What is the probability of head? Is it classical or empirical or subjective probability?
2. A box contains 4 red balls, 6 black balls and 2 white balls. What is the probability of drawing a red ball? Is it classical probability?
3. The director of a nuclear plant feels that the probability of completing the new plant is . What kind of probability is this?
4. A card is drawn randomly from a bridge of deck. What is the probability that it will be i. Queen, ii. Queen of heart, iii. Heart and iv. Heart or spade?
5. A palmist told Fahim of BRAC University that he has a 10 to 3 chance of getting marry this year. What is the probability that Fahim will not get married this year?
6. A balanced die is thrown. What is the probability of getting even number of points on the face? ? Is it classical or empirical or subjective probability?
7. A fair coin is tossed three times. Construct the sample space of the experiment. Hence find: i. A = {At least one tail}, B = {At most three heads}, C = {One head and two tail}, D = {One tail and two heads} and E = {Shows the same face}.
8. For problem 7 find the probability corresponding to the event  and.
9. Among 24 dieters following a similar routine 10 lost weight, 5 gained weight, and 9 remained the same weight, if one of these dieters is randomly chosen find the probability that he or she
10. Gained weight
11. Lost weight
12. Neither lost nor gained weight
13. Two balanced dice are thrown and the number of dots on their upper faces is noted. Write down the sample space for the experiment and determine the probability of the following:
    1. First die shows odd number and second die show even number.
    2. Outcomes of the first die are less than that of second die.
    3. Find the probability that the sum of the outcome is even number.
    4. Find the probability that the sum of the two outcome is greater than 10.

1. Roll two dice and the number of dots on their upper faces is noted. Write down the sample space for the experiment and determine the probability of the following events:
   1. Event A = {Second die shows a number that is not divisible by 3}.
   2. Event B = {Outcome of the first die is divisible by 3}.
   3. Find 
   4. If the sum of the two values is 8, find the probability that one of the values is 3.
   5. Sum of the points on the two dice is 10 or greater if a 5 appears on the first die.
2. A box contains 7 red and 3 black marbles. Three marbles are drawn from the box one after another. Find the probability that the first two are red and the third is black.
3. A box contains 75 marbles, 35 of them are blue and 25 of these blue marbles are swirled. The rest of them are red and 30 of the red ones are swirled. The marbles that are not swirled are clear. What is the probability of drawing:
4. A blue marbles from urn
5. A clear marble from the urn
6. A blue, swirled marble
7. A red clear marble
8. A swirled marble
   1. One card is to be selected at random from an ordinary deck of 52 cards. Find the probability that the selected card is
9. An Ace
10. Not an ace
11. A spade
12. The ace of spade

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* 1. A die is rolled and a coin is tossed, list a sample space and find the probability that the die shows an odd number and the coin shows a head.
  2. The following table lists the 10 countries with the highest production of meat.

|  |  |  |  |
| --- | --- | --- | --- |
| Country | Meat Production (Thousands of metric tons) | Country | Meat Production (Thousands of metric tons) |
| China | 20136 | Brazil | 3003 |
| United States | 17564 | Argentina | 2951 |
| Russia | 12698 | Britain | 2440 |
| Germany | 6395 | Italy | 2413 |
| France | 3853 | Australia | 2373 |

Suppose a world Health Organization committee is formed to discuss the Long – term ramification of producing such quantities of meat. Suppose further that it consists of one representative from each of these countries. If the chair of this committee is then randomly chosen, find the probability that this person will be from a country whose production of meat (in thousands of metric tons)

1. Exceeds 10000
2. Is under 3500
3. Is between 4000 and 6000
4. Is less than 2000

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* 1. If from a standard deck of cards a card is drawn then determine the probability of the following events:
     1. A seven
     2. A black card
     3. An ace or a king
     4. A black 2 or a black 3
     5. A red face card (king, queen or jack)
  2. Two balanced dice, one black and one red are thrown and the number of dots on their upper faces are noted, let b be the outcomes of the black die and r be the outcomes of the red die. Now answer the following:

1. List a sample space of the experiment.
2. What is the probability that the sum is 8, that is?
3. What is the probability that  and ?
4. What is the probability that the number on the red die is at least 4 greater than the number on the black dice.
   1. The blood groups of 200 people is distributed as follows: 50 have type **A** blood, 65 have **B** blood type, 70 have **O** blood type and 15 have type **AB** blood. If a person from this group is selected at random, what is the probability that this person has O blood type?
   2. In a certain class of 30 primary school children there are 16 girls. There are 7 girls and 6 boys with fair hair. A pupil is selected at random to be the class captain. Find the probability that the class captain
5. Is a girl.
6. Is a boy with fair hair.
7. Has not got fair hair.
8. Is a girl and has not got fair hair.
   1. Circle the following options that cannot be a probability?

|  |  |  |
| --- | --- | --- |
| 1. -0.00001 | 1. 0.5 | 1. 1.001 |
|  |  |  |
| 1. 0 | 1. 1 |  |

* 1. A jar contains 3 red marbles, 7 green marbles and 10 white marbles. If three marbles are drawn from the jar one after another without replacement determine the probability of selecting 1 green, 1 red and 1 green marble sequentially.
  2. A customer that goes to the suit department of a certain store will purchase a suit with probability 0.3. The customer will purchase a tie with probability 0.2 and will purchase both a suit and tie with probability 0.1. What proportion of customers purchase neither a suit nor a tie?
  3. Two dice are thrown. Let E be the event that the sum that the sum of the dice is odd, let F be the event that the first die lands on 1, and let G be the event that the sum is 5.

Determine the probability of the event - .

Random Variable & Probability distribution

A variable, whose values are any definite numbers or quantity that arises as a result of chance factors such that they cannot exactly be predicted in advance, is called a random variable.

A random variable is a real – valued function defined over a sample space.

The particular value that the random variable takes on depends upon the outcome of the experiment. That is, we will not know the specific value of the random variable until we have observed the experimental outcome.

# Discrete random variable

A random variable defined over a discrete sample space (*i.e.* that may only take on a finite or countable number of different isolated values) is referred to as a discrete random variable.

Some of the examples are:

1. The number of telephone calls received in a telephone booth during one day;
2. Number of correct answers in 100 – MCQ type examination;
3. Number of defective bulbs produced during a day’s run;
4. Height of nails etc.

**Continues random variable:**

A random variable defined over continues sample space (*i.e.* which may take any vale in a certain interval or collection of intervals), is referred to as a continuous random variable.

Examples of continuous random variable are:

1. Time taken to serve a customer;
2. Weight of a six – month old baby;
3. Average height of students of BRAC University;
4. Temperature recorded by the meteorological office.

**Probability function:**

Any statement of a function associating each of a set of mutually exclusive and exhaustive classes or class intervals with its probability is a probability distribution.

Or in other word –

If  is a discrete random variable with variable with possible values  with corresponding probabilities  then the set of ordered pair  is called the probability function of the discrete random variable .

Symbolically, 

**Properties of probability function:**

If  is probability function of a discrete random variable, then  satisfies the following two properties:

1. , For each possible value of ,
2. 

**Example:**

Let be a random variable with probability function defined as follows

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Values of : | - 2 | 0 | 4 | 11 |
|  | 1/10 | 2/10 | 4/10 | 3/10 |

Find:

|  |  |  |
| --- | --- | --- |
|  |  |  |

**Answer:**

1. = … … … … =
2. = … … … … … … … … … … … = … … … … =
3. = … … … … … … … … … … … = … … … … =

# Assignment

**Problem 1**

A random variable X has the following probability function:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Values of : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|  | a | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 17a |

1. Determine the value of a.
2. Find  and 

**Problem 2:**

A coin is tossed three times in which the probability of head is twice as the probability of tail. If the number of heads is a random variable, find the probability function of the random variable. Also find

|  |  |  |
| --- | --- | --- |
| **a.** | **b.** | **c.** |

**Mathematical Expectation of Random Variable**

**Mathematical expectation of a discrete random variable****:**

If is a discrete random variable which can take finite or infinite sequence of different possible values  with corresponding probabilities then the mathematical expectation of random variable, denoted byis defined by

 - - - - - - - - - - - - - (i)

Sometimes  is called as mathematical expectation of or expected value of or mean of the distribution.

**Problem:**

Find the mean of a random variable having probability function defined as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Values of : | - 2 | 0 | 4 | 11 |
|  | 1/10 | 2/10 | 4/10 | 3/10 |

**Answer:**

|  |  |  |
| --- | --- | --- |
| Mean= | = | 4.7 |

Therefore the mean of the random variable  is 4.7.

**Mathematical expectation of a continuous random variable:**

If is a continuous random variable with probability density function, then the mathematical expectation of X is defined by

 - - - - - - - - - - - - - (ii)

**Problem:**

Suppose that is a continuous random variable with probability function



**Answer:**

According to definition (ii), we have



**Expectation of a function of a random variable:**

Let  be a random variable with probability function  (if is discrete random variable) or density function  (if is continuous random variable). Let be a function of the random variable. Then the mathematical expectation of the random variable  is defined by

--------------------------(iii)

**Problem:**

Let X be a discrete random variable with probability function

|  |  |  |  |
| --- | --- | --- | --- |
| Values of : | 1 | 4 | 9 |
|  | 0.1 | 0.4 | 0.5 |

Find the mean of and .

**Answer:**

By definition mean of  is 

|  |  |  |
| --- | --- | --- |
| Hence the Mean of X= | = | 0.1+1.6+4.5  =6.2 |

Therefore the mean of the random variable  is 6.2.

Calculation of Mean of:

Here 

|  |  |  |
| --- | --- | --- |
| Hence the Mean of | = |  |

Therefore the mean of the random variable  is \_ \_ \_ \_ \_ \_ .

Here 

|  |  |  |
| --- | --- | --- |
| Hence the Mean of | = |  |

Therefore the mean of the random variable  is \_ \_ \_ \_ \_ \_ .

Complete the calculation for Calculation of Mean of:

**Calculation of random variable using expectations:**

Formula for the calculation of random variable using expectation is as follows



**Question:**

Find the variance of the random variable having probability function defined as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Values of : | - 2 | 0 | 4 | 11 |
|  | 1/10 | 2/10 | 4/10 | 3/10 |

**Answer:**

Here,

4.7

Now,

=43.1

Hence variance of is

43.1 – 22.09=21.01

And the standard deviation is

 =

Problem:

A random variable with probability function defined by

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Values of X:x |  | 1 | 2 | 3 |
|  |  |  |  |  |

1. Determine the mean and variance for the given information.
2. Determine the probability that the value of the variable is fewer than 2.

Solution:

1. Calculation of the mean from using the probability function:

|  |  |  |
| --- | --- | --- |
| Mean of, |  |  |
|  |  |  |

Calculation of the variance by using the probability function:

Variance of,

Now







Also we already found that 

Hence Variance of, 

And the standard deviation 

1. the probability that the value of the variable is fewer than 2

*i.e.* =

**Problems**

1. Hospital records indicated that knee replacement patients stayed in the hospital for the number of days shown in the distribution

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Number of days Stayed | 3 | 4 | 5 | 6 | 7 |
| Frequency | 15 | 32 | 56 | 19 | 5 |

Find these probabilities:

1. A patient stayed exactly stayed 4 days
2. A patient stayed less than 5 days
3. A patient stayed at most 3 days
4. A patient stayed at least 3 days
5. A bank vice president feels that each savings account customer has, on average, three credit cards. The following distribution represents the number of credit cards people own. Find the mean, variance, and standard deviation. Is the vice president correct?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Number of cards (X) | 0 | 1 | 2 | 3 | 4 |
| Probability ***P(X)*** | 0.18 | 0.44 | 0.27 | 0.08 | 0.03 |

**Answer: 1.3; 0.9; 1.0; No, on average each person has about 1 credit card / BLUMAN 267**

1. The number of suits sold per day at a retail store is shown in the table, with the corresponding probabilities.
2. Find the mean and standard deviation of the distribution.
3. If the manager of the retail store wants to be sure that he has enough suits for the next 5 days, how many should the manager purchase?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Number of suits sold : | 19 | 20 | 21 | 22 | 23 |
| Probability *P*(*X)* | 0.2 | 0.2 | 0.3 | 0.2 | 0.1 |

**Answer: 20.8; 1.6; 1.3 / BLUMAN 267**

1. A study conducted by a TV station showed the number of televisions per household and the corresponding probabilities for each. Find the mean, variance, and standard deviation. If you were taking a survey on the programs that were watched on television, how many program diaries would you send to each household in the survey?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Number of television : | 1 | 2 | 3 | 4 |
| Probability *P*(*X)* | 0.32 | 0.51 | 0.12 | 0.05 |

**Answer: 1.9; 0.6; 0.8; 2 Diaries / BLUMAN 5-17**

1. From past experience, a company found that in cartons of DVDs, 90% contain no defective DVDs, 5% contain one defective DVD, 3% contain two defective DVDs, and 2% contain three defectives DVDs. Find the mean, variance, and standard deviation for the number of defective DVDs.

**Answer: 0.17; 0.321; 0.567 / BLUMAN 5-17**

1. The probabilities that a player will get 5 to 10 questions right on a trivia quiz are shown below. Find the mean, variance, and standard deviation for the distribution..

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Number of corrects answer (X) | 5 | 6 | 7 | 8 | 9 | 10 |
| Probability ***P(X)*** | 0.05 | 0.2 | 0.4 | 0.1 | 0.15 | 0.1 |

**Answer: 7.4; 1.84; 1.356 / BLUMAN 5-17**

1. A coin is tossed three times in which the probability of head is twice as the probability of tail. If the number of heads is a random variable, determine the probability function of the random variable. Calculate –
2. and
3. and V(2x+5)
4. The pressure measured in pounds per cm2 at a certain valve is a random variable X whose probability density function is

=

Find the probability that the pressure at this valve is

1. not more than 2 pounds per cm2
2. greater than 2 pounds per cm2
3. between 1.5 and 2.5 pounds per cm2
4. less than 1.5 pounds per cm2

**Conditional Probability: Total Probability theorem & Bayes theorem**

**Conditional Probability:**

If A and B are two events is a probability space A,  ), such that , then the conditional probability of A given B, denoted by  is defined by



Similarly the conditional probability of B given A, denoted by  is defined by



**Theorem of total probabilities:**

Let be a sequence of n mutually exclusive events in A of a given probability space A,  ) such that  and for all then for any event  A



**Bayes Theorem:**

Let be a sequence of n mutually exclusive events of a given probability space, such that and for all  and if A is an event satisfying then the according to bayes theorem



**Problem related to Bayes theorem and theorem of total probability:**

**Problem 1:**

Mr. Khandker wants to build a house this year he applied for a bank loan. The probability that he will get it is. If he will get the bank loan, the probability that he will build the house is. However, if he will not get the bank loan, the probability that he will build the house is. What is the probability that Mr. Khandker will build the house this year?

**Solution:**

Let us define the following events

 Mr. Khandker will get the loan

 Mr. Khandker will not get the loan

 He will build the house

We are given

, 

and 

We need to find the probability of building the house in the following year. By applying the theorem of total probability we have

= 

**Problem 2:**

Two sets of candidate are competing for the position on the Board of Directors of a company. The probabilities that the first and second sets will win are 0.6 and 0.4, respectively. If the first set wins the probability of introducing a new product with in a month is 0.8 and the corresponding probability if the second set win is 0.3. What is the probability that the new product will be introduced with in a month?

Solution:

Let us define the following events

 The 1st set will win

The 2nd set will win

The new product will be launched with in a month

We are given

, 

and 

We need to find the probability of introducing the new product with in a month of the election. By applying the theorem of total probability we have

= =

**Problem 3:**

A certain disease is present in about 1 out of 1000 persons in a given population. Suppose that there is simple blood test which gives a positive reading with probability 0.99 for a diseased person and with probability 0.05 for a healthy person. A person is selected at random from this population, what is the probability that the blood test of the person will give positive reading? Also if the blood test of a selected person gives positive reading what the probability that he does have the disease is?

Solution:

Let us define the following events

 The person selected at random from the population has the disease

 The Person selected at random from the population does not have the disease

The blood test will be positive

We are given

, 

and 

We need to find the probability of that the test will be positive. By applying the theorem of total probability we have

= =

Now if the blood test is positive then we need to find the probability that the actually has the disease



**Problem 4:**

Three persons A, B and C are being considered for the appointment as Vice-Chancellor of a University whose chance4s of being selected fro the post are in the proportion 5:3:2 respectively. The probability the A, if selected will introduce democratization in the University structure is 0.3, the corresponding probabilities for B and C doing the same are respectively 0.6 and 0.8. What is the probability that democratization would be introduced in the University?

**Solution:**

Let us define the following events

 A will be selected as Vice Chancellor

 B will be selected as Vice Chancellor

 C will be selected as Vice Chancellor

 Selected vice chancellor will introduce democratization in the university

We are given

, and 

,  and 

We need to find the probability that the democratization process will be introduces in the university system. By applying the theorem of total probability we have

=

**Problem 5:**

In a bolt company machine A produces 45% of the output and machine B produces the rest. On the average machine B produces the rest. On the average 9 items in 1000 produced by machine A are defective and 2 items in 500 produced by B are defective. In a day’s run, the two machines produce 20,000 items. An item is drawn at random from a day’s output and is found to be defective. What is the probability that it was produced by machine A? Also calculate that it was produced by B?

Solution:

Let us define the following events

 Item produced by machine A

 Item produced by machine B

 Selected item is defective

We are given

, 

and 

We need to find and 

According to Bayes theorem,



According to total probability theorem



Therefore,





**Problem 6:**

In a course 65% students are female. The probability that a female student passes the course is 0.8 and the probability that a male student passes the course is 0.75. A student is selected at random from this class and is found to be passed. What is the probability that the student is a female student?

**Solution:**

Let us define the following events

 The student selected at random from the class is male

 The student selected at random from the class is male

The student passes the course

We are given

, 

and 

We need to find the probability of that the selected student passed. By applying the theorem of total probability we have

=

Now if the selected student passed then we need to find the probability that the selected student is female



**Problem 7:**

Moushumi, Sharmina and Aparna are there sisters. They do the family job 40%, 30% and 30% respectively. The probability that one dish will be broken by Moushumi when she is washing them is 0.02, for Shamina and Aparna the probabilities are 0.03 and 0.02, respectively. One night the parents hear one break but they don’t know who is washing them. What is the probability that Moushumi washing them?

Solution:

Let us define the following events

 Moushumi does the job

 Sharmina does the job

 Aparna does the job

Broken dish

We are given

,  and 

, and 

We need to find the probability of that if there is a break it is done by Moushumi,  

According to theorem of total probability we have

=

Now if there is a break the probability that it was done by Moushumi



**Problem 8:**

Dr. Shobahani diagnoses cancer correctly 80% cases. The chance that a patient will die by his treatment after diagnosis is 30% and the chance of death by wrong diagnosis is 90%. A patient of Dr. Shobahani who had cancer died. What is the probability that his diagnosis was wrong?

Solution:

Let us define the following events

 Cancer was correctly diagnosed

 Cancer was not correctly diagnosed

 A patient of Dr. Salem who had cancer died

We are given

 and 

, and 

A patient died and we need to find that the diagnosis was wrong  

According to theorem of total probability we have

=

Now given a patient died we need to find the the probability that the diagnosis was wrong



**Problem 9:**

Below are listed the numbers of doctors in various specialties by gender.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Pathology | Pediatrics | Psychiatry |
| Male | 12,575 | 33,020 | 27,803 |
| Female | 5,604 | 33,351 | 12,292 |

One doctor is chosen at random.

1. Find .
2. Find

**Binomial distribution**

A discrete random variable  is said to have a binomial distribution if its probability function is given by



**Binomial Experiment:**

A random experiment is said to be a binomial experiment if

1. There are a fixed number of trials (n trials)
2. In each trial there are only two possible outcomes “success” and “failure”
3. The probability of success (p) and the probability of failure (q) remain same from trial to trial.

**Remarks**

1. The mean of the binomial distribution is.
2. The variance of the binomial distribution is.

**Example:**

In a community the probability that a newly born child will be boy is. Among 4 newly born children in that community. What is probability that

1. All are boys,
2. At least two boys
3. No boys
4. Exactly one boy
5. At most two boys

**Answer:**

Here

Number of success that is the newly born child will be a boy.

 Probability of the newly born child will be boy = 

 Probability of the newly born child will be girl = 

a. [All boys]… … … … … =

b. [At least 2 boys]

= = … … … … = 

c. [No boys] … … =

d. [Exactly one boy] … … =

e. [At most 2 boys]= … … … … =

**Answer:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| a. | b. | c. | d. | e. |

# Assignment

1. A fair coin is tossed 5 times. Find the probability of –

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1. Exactly two heads | | | 1. At least 3 heads | | | 1. No heads | | |
| 1. At most 2 heads. | | | 1. Exactly 4 heads | | | 1. Fewer than 3 heads | | |
| **Answer:** |  |  | |  |  | |  |  |

1. If is a binomial variable with  and ; evaluate the following

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | |  | | | |  | |
|  | | | | |  | | | |
| **Answer:** | 1. 0.9977 | 1. 0.3087 | | 1. 0.8370 | | 1. 0.4717 | | 1. 0.1630 |

1. The probability that a patient recovers from a delicate heart operation is. What is the probability that exactly five of the next seven patients undergoing this operation survive?

***(Answer: 0.124)***

1. Suppose a poll of 20 voters is taken in a large city. The purpose is to determine the numbers who favor a certain candidate for mayor. Suppose that 60% of all the city’s voters favor the candidate.

Find the following:

* + 1. Find the mean and standard deviation of. **Answer:.**
    2. Find the probability that. **Answer: **
    3. Find the probability that. **Answer: **
    4. Find the probability that. **Answer: **

1. If  is a binomial random variable, compute for each of the following cases:
2. ; **Answer: **
3. ; **Answer: **
4. ; **Answer: **
5. 10% of men and 0.25% of women cannot distinguish between the colors red and green. This is the type of color blindness that causes problems with traffic signals. If 6 men are randomly selected from a study of traffic signal perception, find the probability that exactly 2 of them cannot distinguish between red and green.
6. A survey database shows that 35 percent of U.S. adults indicate that they have been tested for HIV at some point in their life. Consider a simple random sample of 15 adults selected at that time. Find the probability that the number of adults who have been tested for HIV in the sample would be:
7. Three
8. Less than five
9. Between five and nine, inclusive

**Poisson distribution**

* Poisson distribution is a limiting case of the binomial distribution under the following conditions:
  1. The probability of success or the probability failure  in any trial is very small, that is  or .
  2. The number of trial  is very large, that is 
  3.  (say) is a finite constant.
     + Poisson distribution:

A discrete random variable  is said to have a Poisson distribution if its probability function is given by



**Some examples where Poisson distribution may be successively applied:**

* + - 1. The number of cars passing through a certain street in time t.
      2. Number of suicide reported in a particular day.
      3. Number of faulty blades in a packet of 100.
      4. Number of printing mistakes at each page of a book.
      5. Number of air accidents in some unit of a book.

**Remarks**

1. The mean of the Poisson distribution is.
2. The variance of the Poisson distribution is.

## Assignment

* 1. Suppose that the number of emergency patients in a given day at a certain hospital is a Poisson variable  with parameter. What is the probability that in a given day there will be -

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1. 15 emergency patients | | | 1. At least 3 emergency patients | |
| 1. More than 20 but less than 25 patients. | | | | |
| **Answer:** | 1. 0.0516 | 1. 1 | | 1. 0.2441 |

* 1. If the probability that a car accident happens in a very busy road in an hour is 0.001. if 2000 cars passed in an hour by that road, what is the probability that

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1. Exactly 3 | 1. More than 2 car accidents | **Answer:** | 1. 0.180 | 1. 0.323 |

* 1. Assume that  is a random variable having a Poisson probability distribution with a mean of 1.5. Use Poisson distribution table to find the following probabilities.

a.; Answer: 0.934

b.; Answer: 0.191

* 1. In a study of the effectiveness of an insecticide against a certain insect, a large area of land was sprayed. Later the area was examined for live insects by randomly selecting squares and counting the number of live insects per square. Past experience has shown the average number of live insects per square after spraying to be If the number of live insects per square follows a Poisson distribution, find the probability that a selected square will contain:

1. No live insects
2. Exactly four live insects

**Note: For calculating probabilities of Binomial & Poisson distribution students have to see “Table of Binomial distribution” & “Table of Poisson distribution”. Students must arrange this table during the quiz and final examination by them selves.**

Normal distribution

Normal distribution is the most important probability distribution in statistics. The distributions of heights, weights, errors made in measuring certain physical quantities are just a few among the countless measurements whose distributions are normal.

**Normal Variable and Distribution:**

A continuous random variable  is said to be a normal variable if its probability distribution function is given by

, 

Where the parameters  and are the mean and variance of the distribution and satisfy the conditions  and.

A normal variable, with mean  and variance is denoted by .

**Standard Normal Variable and Distribution:**

If  is a normal variable with parameters and then  is known as standard normal variable with mean zero and variance unity.

The probability distribution of a standard normal variable is known as standard normal distribution and is denoted by.

, 

**Assignment 1:** Let be a standard normal random variable. Find the probability that  will be

|  |  |  |
| --- | --- | --- |
| 1. Less than 1.5 | 1. Greater than 2.4 | 1. Between 1.5 and 2.14 |
| 1. Less than - 1.32 | 1. Greater than –2.52 | 1. Between – 2.52 and 1.64 |
| 1. Between - 2.52 and – 1.5 | 1. Exactly 2 |  |

**Answer:**

|  |  |  |  |
| --- | --- | --- | --- |
| 1. 0.9332 | 1. 0.0082 | 1. 0.0506 | 1. 0.0934 |
| 1. 0.9941 | 1. 0.9436 | 1. 0.0609 | 1. 0 |

**Assignment 2:**

Suppose that the average temperature in July in a certain region is a normal random variable with parameters  (measured in degrees Fahrenheit) and. Find the probability that in a given year the average temperature in July in that region will be

|  |  |
| --- | --- |
| 1. Above | 1. Below |
| 1. Between  and | 1. Below |

**Answer:**

|  |  |
| --- | --- |
| 1. 0.00228 | 1. 0.8413 |
| 1. 0.6826 | 1. 0.3446 |

**Assignment 3:**

A certain type of insect survives on the average 3.0 years, with a variance of 0.25 year. Assuming that the lives of the insect are normally distributed, find the probability that a given insect will survive less than 2.3 years. **(Answer: 0.0808)**

**Assignment 4:**

The grade - point average score of 80 student of Department of Computer Science of Dhaka University in their term final examination was found to follow approximately a normal distribution with a mean of 2.1 and a standard deviation 0.6. How many of the students are expected to have a score between 2.5 and 3.5? **(Answer: 20)**

**Assignment 5:**

The Philips Bangladesh manufactures electric bulbs that have length of life that is normally distributed with mean equal to 800 hours and standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. **(Answer: 0.5111)**

**Assignment 6:**

A continuous manufacturing process produces items whose weights are normally ,distributed with a mean weight of 800 gms and a standard deviation of 300 gms. A random sample of 16 items is to be drawn from the process. What is the probability that the arithmetic mean of the sample exceeds 900

**(Answer: \*\*\*\*)**

**Assignment 7:**

The weights of a certain population of young adult females are approximately normally distributed with a mean of 132 pounds and a standard deviation of 15. Find the probability that a subject at from this population weigh:

1. more than 155 pounds
2. between 105 and 145 pounds

**Assignment 8:**

Suppose the average length of stay in a chronic disease hospital of a certain type of patient is 60 days with a standard deviation of 15. If it is reasonable to assume an approximately normal distribution of lengths of stay, find the probability that a randomly selected patient from this group will have a length of stay:

1. Greater than 50 days
2. Between 30 and 60 days
3. Greater than 90 days

**Assignment 9:**

X is a normally distributed variable with mean μ = 30 and standard deviation σ = 4. Find   
a) P(x < 40)   
b) P(x > 21)   
c) P(30 < x < 35)

**Assignment 10:**

A radar unit is used to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr. What is the probability that a car picked at random is travelling at more than 100 km/hr?

**Assignment 11:**

For a certain type of computers, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. John owns one of these computers and wants to know the probability that the length of time will be between 50 and 70 hours.

**Assignment 12:**

Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Tom takes the test and scores 585. Will he be admitted to this university?

**Assignment 13:**

The length of similar components produced by a company are approximated by a normal distribution model with a mean of 5 cm and a standard deviation of 0.02 cm. If a component is chosen at random

* 1. what is the probability that the length of this component is between 4.98 and 5.02 cm?
  2. what is the probability that the length of this component is between 4.96 and 5.04 cm?

**Assignment 13:**

The length of life of an instrument produced by a machine has a normal ditribution with a mean of 12 months and standard deviation of 2 months. Find the probability that an instrument produced by this machine will last   
a) less than 7 months.   
b) between 7 and 12 months.

**Assignment 14:**

The time taken to assemble a car in a certain plant is a random variable having a normal distribution of 20 hours and a standard deviation of 2 hours. What is the probability that a car can be assembled at this plant in a period of time

1. less than 19.5 hours?
2. between 20 and 22 hours?

**Assignment 14:**

A large group of students took a test in Physics and the final grades have a mean of 70 and a standard deviation of 10. If we can approximate the distribution of these grades by a normal distribution, what percent of the students

1. scored higher than 80?
2. should pass the test (grades≥60)?
3. should fail the test (grades<60)?

**Assignment 15:**

The annual salaries of employees in a large company are approximately normally distributed with a mean of $50,000 and a standard deviation of $20,000.

1. What percent of people earn less than $40,000?
2. What percent of people earn between $45,000 and $65,000?
3. What percent of people earn more than $70,000?

**Note: For calculating probabilities of Normal distribution students have to consult with the “Table of Standard normal distribution”. Students must arrange this table during the quiz and final examination by themselves.**